

# Chapter 10

## MAT FOUNDATIONS

### Omitted parts:

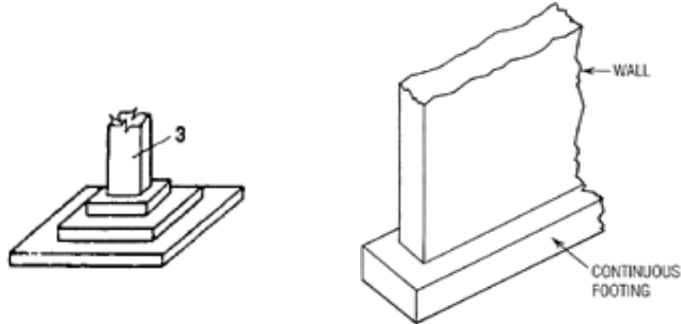
Sections 10.5, 10.6

Example 10.8

# Types of shallow foundations

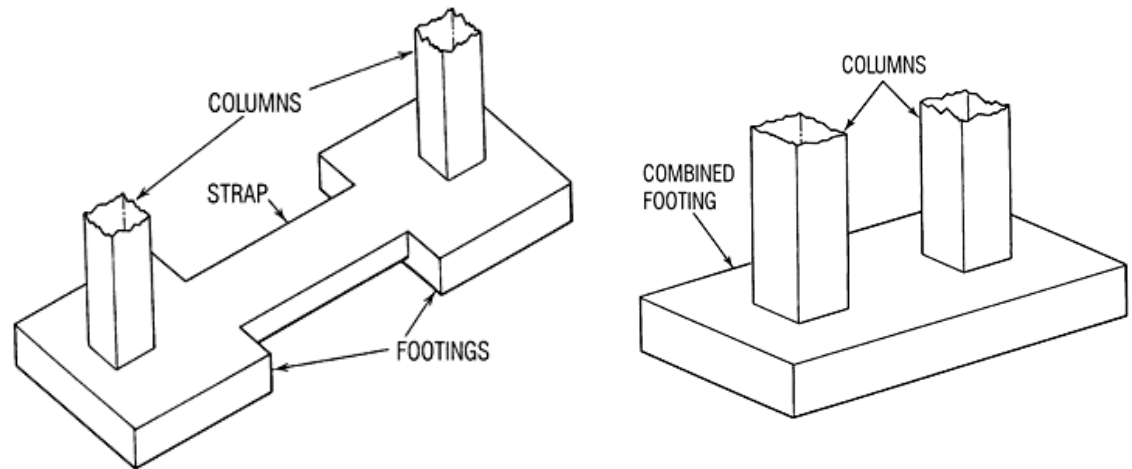
1

Spread



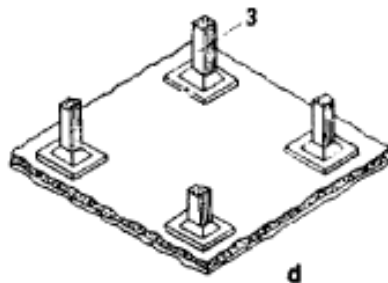
2

Combined



3

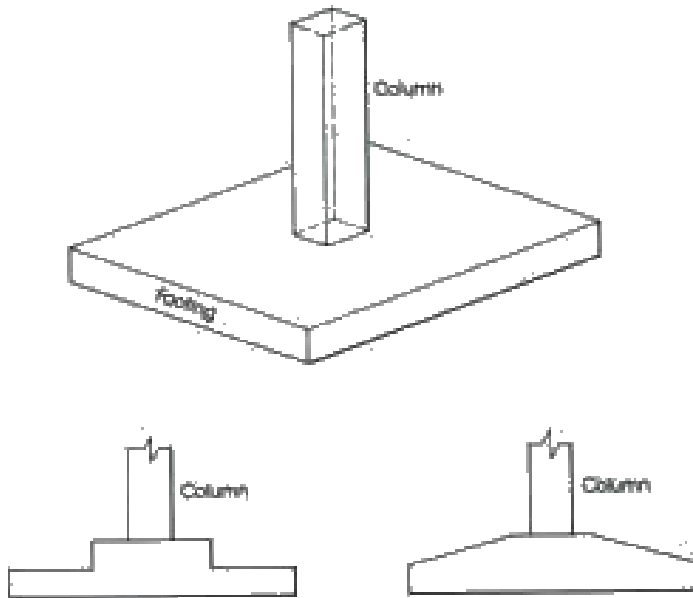
Mat (Raft)



# SPREAD FOOTINGS

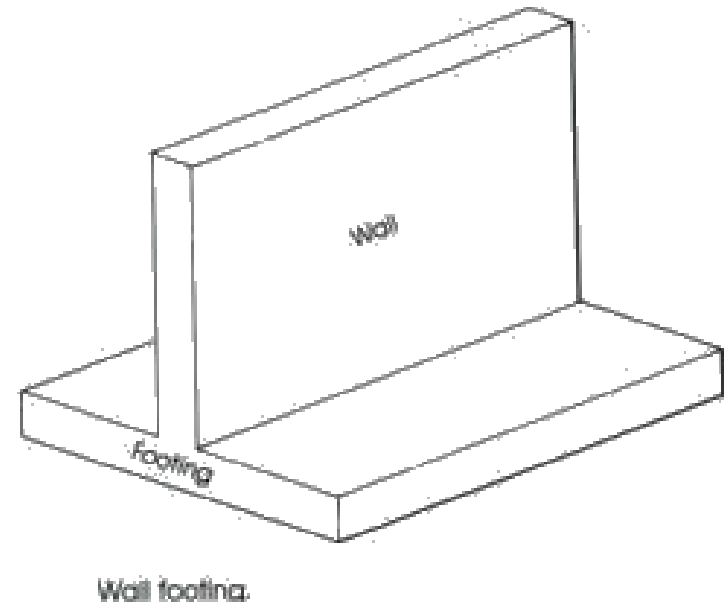
## Pad Foundations

often rectangular or square and are used to support single columns. This is one of the most economical types of footings and is used when columns are spaced at relatively long distances.



## Strip Foundation

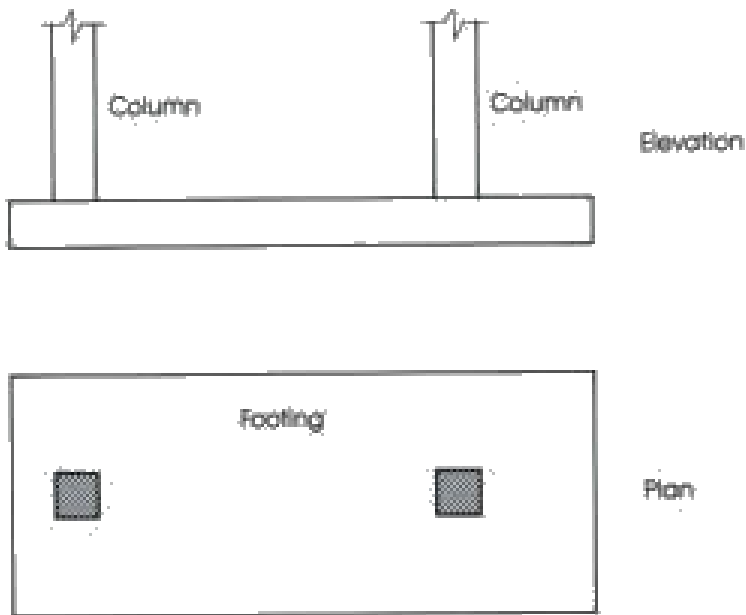
Strip footings are **continuous** foundation used to support **walls**.



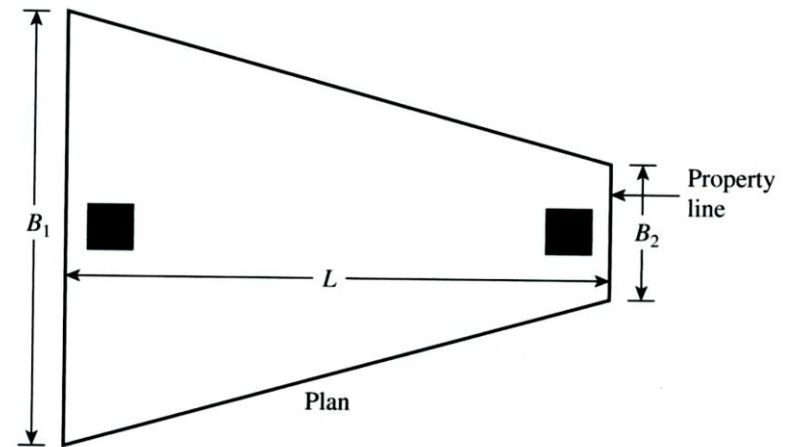
# COMBINED FOOTINGS

Combined footings are used when two columns are so close that single footings cannot be used or when one column is located at or near a property.

## 1. Rectangular Combined Footing



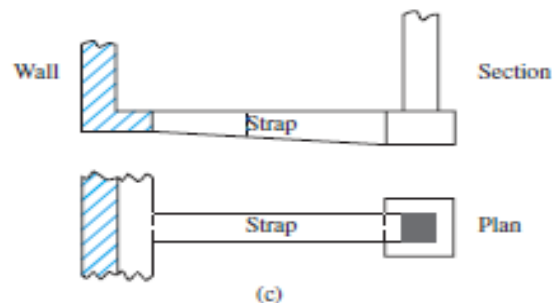
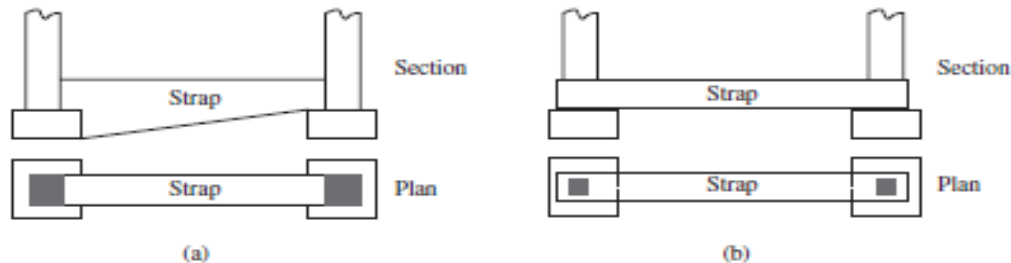
## 2. Trapezoidal Combined Footing



# COMBINED FOOTINGS

## 3. Cantilever Footings

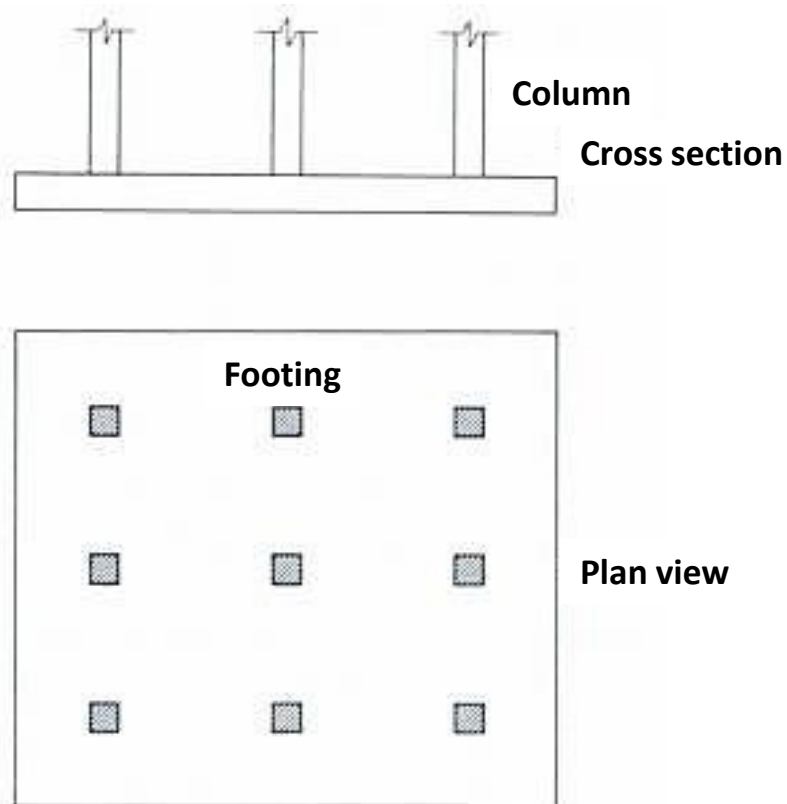
- ❑ Cantilever footing construction uses a *strap beam* to connect an eccentrically loaded column foundation to the foundation of an interior column.
- ❑ Cantilever footings may be used in place of trapezoidal or rectangular combined footings when the allowable soil bearing capacity is high and the distances between the columns are large.
- ❑ It consists of two single footings connected with a **beam** or a strap and support two single columns. This type replaces other combined footings and is more economical.



# Mat (Raft) Foundations

## Mat (Raft) Foundations

Consists of one slab usually placed under the entire building area.



# COMBINED FOOTINGS

Combined footings can be classified generally under the following categories:

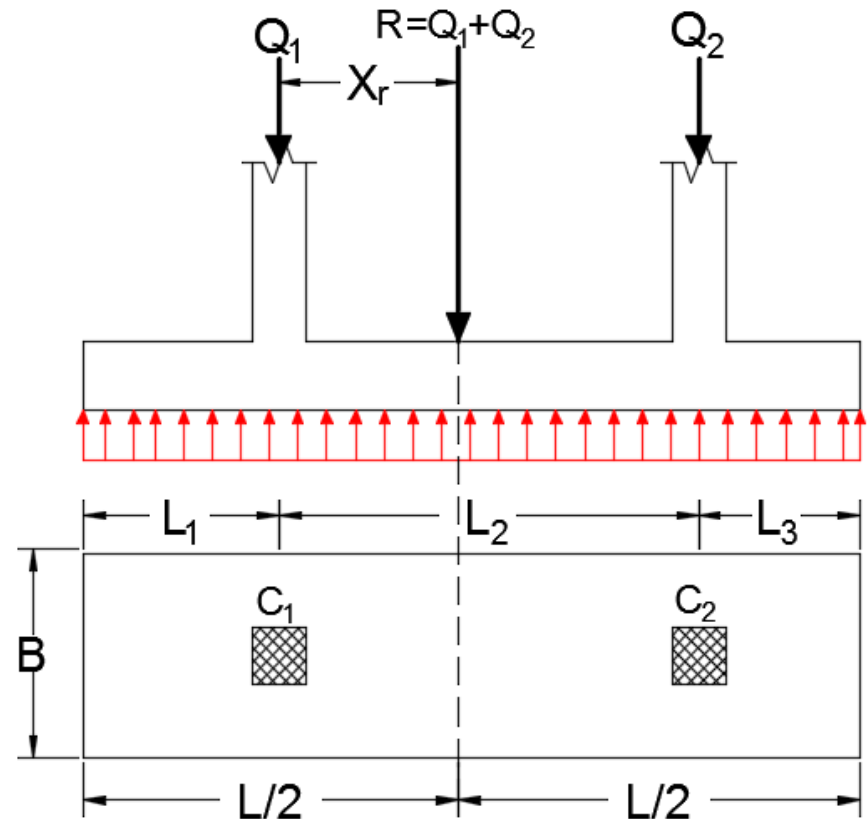
- Rectangular combined footing
- Trapezoidal combined footing
- Cantilever (strap) footing

# RECTANGULAR COMBINED FOOTINGS

There are three cases:

## 1. Extension is permitted from both side of the footing

To keep the pressure under the foundation uniform, the resultant force of all columns loads ( $R$ ) must be at the center of the footing, and since the footing is rectangular,  $R$  must be at the middle of the footing (at distance  $L/2$ ) from each edge to keep uniform pressure.

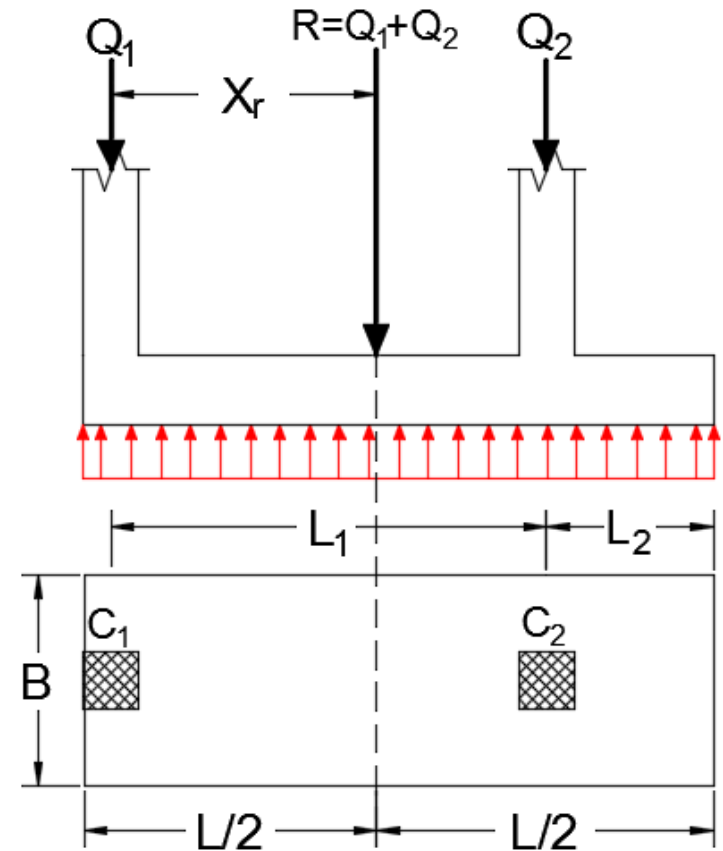




# RECTANGULAR COMBINED FOOTINGS

## 2. Extension is permitted from one side and prevented from other side:

The only difference between this case and case 1 that the extension exists from one side and when we find  $X$  we can easily find  $L$ :  
To keep the pressure uniform  
 $X + \text{column width}/2 = L/2$ .



# RECTANGULAR COMBINED FOOTINGS

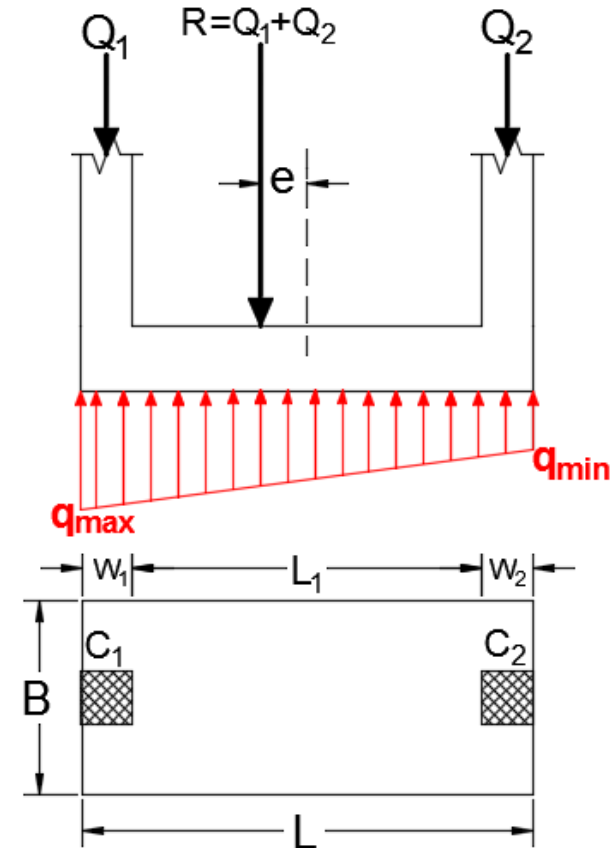
## 3. Extension is not permitted from both sides of the footing:

In this case the resultant force  $R$  is not at the center of rectangular footing because  $Q_1$  and  $Q_2$  are not equals and no extensions from both sides. So the pressure under the foundation is not uniform and we design the footing in this case as following:  $L=L_1+W_1+W_2$

How we can find  $e$ :

$$\sum M_{\text{foundation center}} = 0.0$$

$$Q_1 \times \left( \frac{L}{2} - \frac{W_1}{2} \right) - Q_2 \times \left( \frac{L}{2} - \frac{W_2}{2} \right) = R \times e$$



# RECTANGULAR COMBINED FOOTINGS

## 3. Extension is not permitted from both sides of the footing:

The eccentricity in the direction of L:

Usually  $e < \frac{L}{6}$  (because L is large)

$$q_{\max} = \frac{R}{B \times L} \left( 1 + \frac{6e}{L} \right)$$

$$q_{\text{all,gross}} \geq q_{\max} \rightarrow q_{\text{all,gross}} = q_{\max} \text{ (critical case)}$$

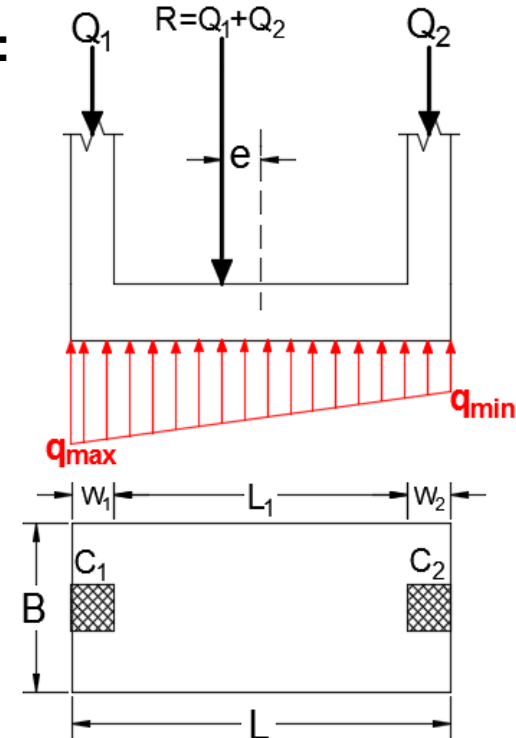
$$q_{\text{all,gross}} = \frac{R}{B \times L} \left( 1 + \frac{6e}{L} \right) \rightarrow B = \checkmark .$$

### Check for B:

$$q_{\min} = \frac{R}{B \times L} \left( 1 - \frac{6e}{L} \right) \text{ must be } \geq 0.0$$

If this condition doesn't satisfied, use the modified equation for  $q_{\max}$  to find B:

$$q_{\max,\text{modified}} = \frac{4R}{3B(L - 2e)} \rightarrow B = \checkmark .$$



# RECTANGULAR COMBINED FOOTINGS

- a. Determine the area of the foundation

$$A = \frac{Q_1 + Q_2}{q_{\text{net(alt)}}}$$

where

$Q_1, Q_2$  = column loads

$q_{\text{net(alt)}}$  = net allowable soil bearing capacity

- b. Determine the location of the resultant of the column loads. From Figure :

$$X = \frac{Q_2 L_3}{Q_1 + Q_2}$$

- c. For a uniform distribution of soil pressure under the foundation, the resultant of the column loads should pass through the centroid of the foundation. Thus,

$$L = 2(L_2 + X)$$

where  $L$  = length of the foundation.

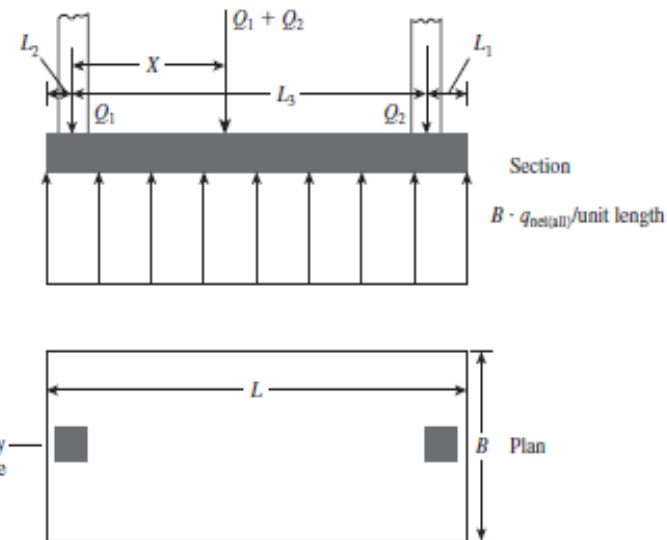
- d. Once the length  $L$  is determined, the value of  $L_1$  can be obtained as follows:

$$L_1 = L - L_2 - L_3$$

Note that the magnitude of  $L_2$  will be known and depends on the location of the property line.

- e. The width of the foundation is then

$$B = \frac{A}{L}$$



# EXAMPLE 10.1

Refer to Figure 8.1. Given:

$$Q_1 = 400 \text{ kN}$$

$$Q_2 = 500 \text{ kN}$$

$$q_{\text{net(alt)}} = 140 \text{ kN/m}^2$$

$$L_3 = 3.5 \text{ m}$$

Based on the location of the property line, it is required that  $L_2$  be 1.5 m. Determine the size ( $B \times L$ ) of the rectangular combined footing.

## Solution

Area of the foundation required is

$$A = \frac{Q_1 + Q_2}{q_{\text{net(alt)}}} = \frac{400 + 500}{140} = 6.43 \text{ m}^2$$

Location of the resultant [Eq. (8.2)] is

$$X = \frac{Q_2 L_3}{Q_1 + Q_2} = \frac{(500)(3.5)}{400 + 500} \approx 1.95 \text{ m}$$

For uniform distribution of soil pressure under the foundation from Eq. (8.3), we have

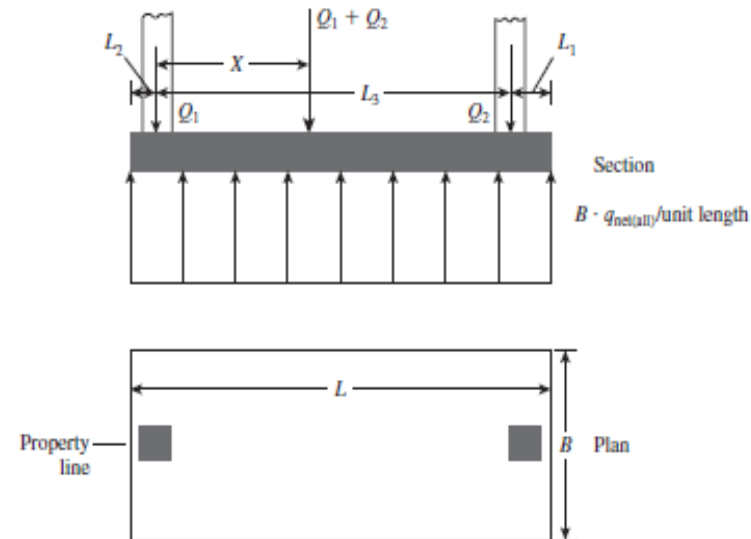
$$L = 2(L_2 + X) = 2(1.5 + 1.95) = \mathbf{6.9 \text{ m}}$$

Again, from Eq. (8.4),

$$L_1 = L - L_2 - L_3 = 6.9 - 1.5 - 3.5 = 1.9 \text{ m}$$

Thus,

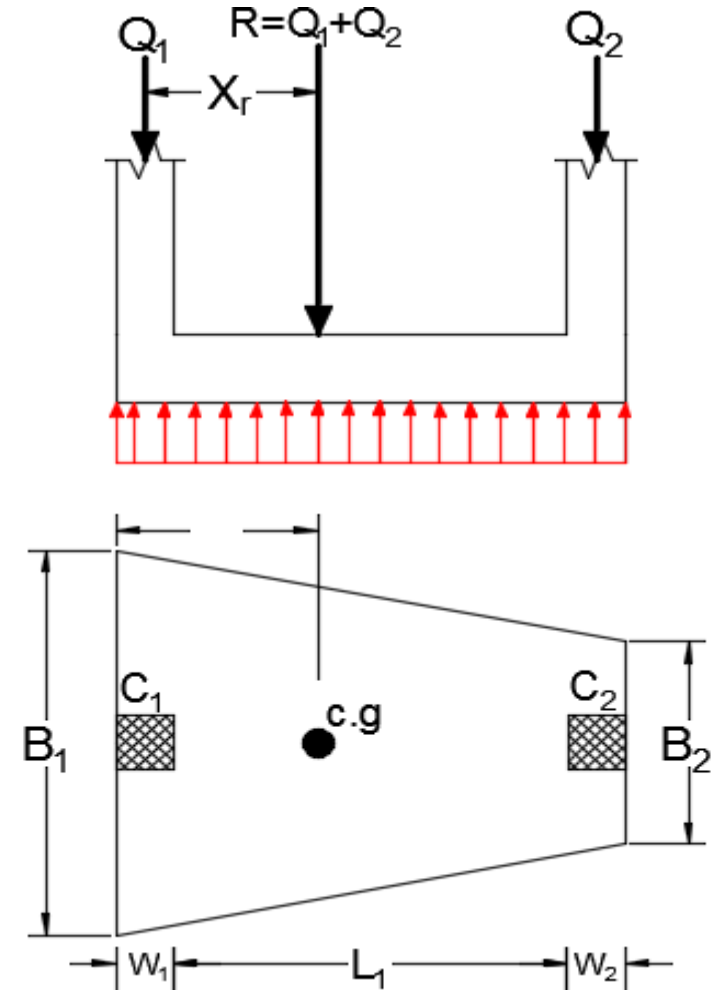
$$B = \frac{A}{L} = \frac{6.43}{6.9} = \mathbf{0.93 \text{ m}}$$



# TRAPEZOIDAL COMBINED FOOTINGS

## Advantages:

1. More economical than rectangular combined footing in case of “extension is not permitted from both sides” especially if there is a large difference between columns loads.
2. To keep uniform contact pressure in case of “extension is not permitted from both sides”, use trapezoidal footing because the resultant force “R” can be located at the centroid of trapezoidal footing.



# TRAPEZOIDAL COMBINED FOOTINGS

## Design:

$Q_1 > Q_2 \rightarrow B_1$  at  $Q_1$  and  $B_2$  at  $Q_2$

$L = L_1 + W_1 + W_2$

$$A_{\text{req}} = \frac{\sum Q_{\text{service}}}{q_{\text{all,net}}} = \frac{Q_1 + Q_2}{q_{\text{all,net}}}$$

$$\frac{Q_1 + Q_2}{q_{\text{all,net}}} = \frac{L}{2} (B_1 + B_2) \quad (1)$$

Now take summation moments at  $C_1$  equals zero to find  $X_r$ :

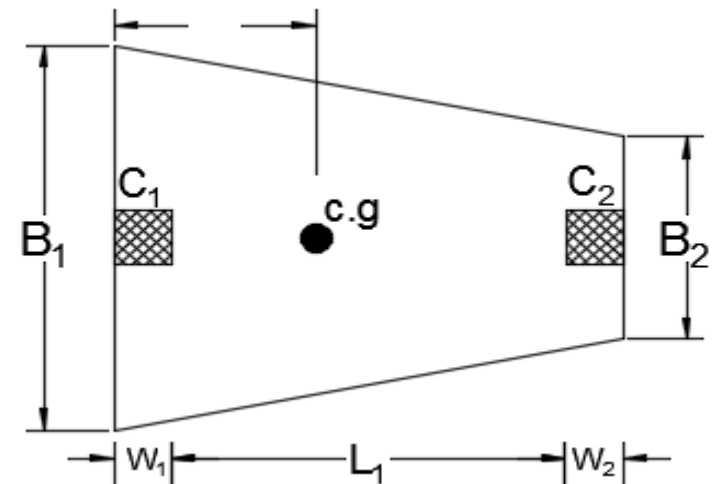
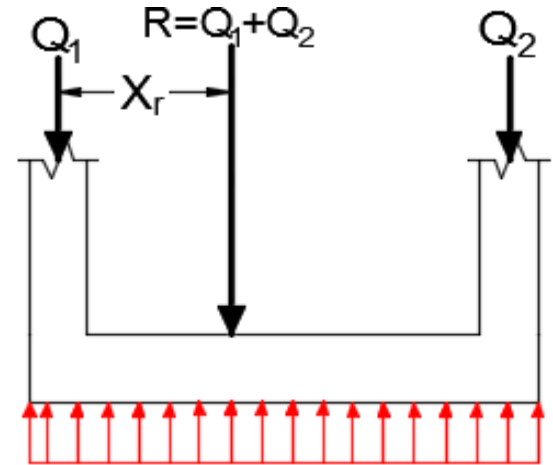
$$\sum M_{C_1} = 0.0 \rightarrow Q_2 L_1 + (W_f + W_s) \times X_r = R \times X_r$$

$$X_r + \frac{W_1}{2} = \bar{X}$$

$$\bar{X} = \frac{L}{3} \left( \frac{B_1 + 2B_2}{B_1 + B_2} \right) \quad (2)$$

Solve Eq. (1) and Eq. (2)

$B_1 = \checkmark$  and  $B_2 = \checkmark$ .



# TRAPEZOIDAL COMBINED FOOTINGS

- a. If the net allowable soil pressure is known, determine the area of the foundation:

$$A = \frac{Q_1 + Q_2}{q_{\text{net(alt)}}}$$

From Figure 8.2,

$$A = \frac{B_1 + B_2}{2} L$$

- b. Determine the location of the resultant for the column loads:

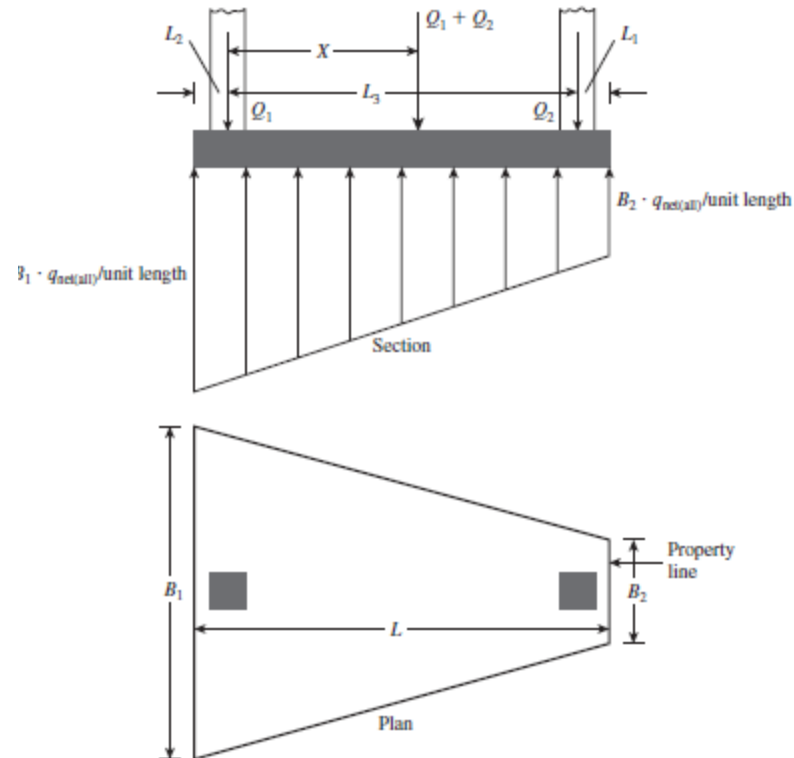
$$X = \frac{Q_2 L_3}{Q_1 + Q_2}$$

- c. From the property of a trapezoid,

$$X + L_2 = \left( \frac{B_1 + 2B_2}{B_1 + B_2} \right) \frac{L}{3}$$

With known values of  $A$ ,  $L$ ,  $X$ , and  $L_2$ , solve Eqs. (8.7) and (8.9) to obtain  $B_1$  and  $B_2$ . Note that, for a trapezoid,

$$\frac{L}{3} < X + L_2 < \frac{L}{2}$$





# EXAMPLE 10.2

Refer to Figure 8.2. Given:

$$Q_1 = 1000 \text{ kN}$$

$$Q_2 = 400 \text{ kN}$$

$$L_3 = 3 \text{ m}$$

$$q_{\text{net(alt)}} = 120 \text{ kN/m}^2$$

Based on the space available for construction, it is required that  $L_2 = 1.2 \text{ m}$  and  $L_1 = 1 \text{ m}$ . Determine  $B_1$  and  $B_2$ .

### Solution

The area of the trapezoidal combined footing required is [Eq. (8.6)]

$$A = \frac{Q_1 + Q_2}{q_{\text{net(alt)}}} = \frac{1000 + 400}{120} = 11.67 \text{ m}^2$$

$$L = L_1 + L_2 + L_3 = 1 + 1.2 + 3 = 5.2 \text{ m}$$

From Eq. (8.7),

$$A = \frac{B_1 + B_2}{2} L$$

$$11.67 = \left( \frac{B_1 + B_2}{2} \right) (5.2)$$

or

$$B_1 + B_2 = 4.49 \text{ m} \quad (a)$$

From Eq. (8.8),

$$X = \frac{Q_2 L_3}{Q_1 + Q_2} = \frac{(400)(3)}{1000 + 400} = 0.857 \text{ m}$$

Again, from Eq. (8.9),

$$X + L_2 = \left( \frac{B_1 + 2B_2}{B_1 + B_2} \right) \frac{L}{3}$$

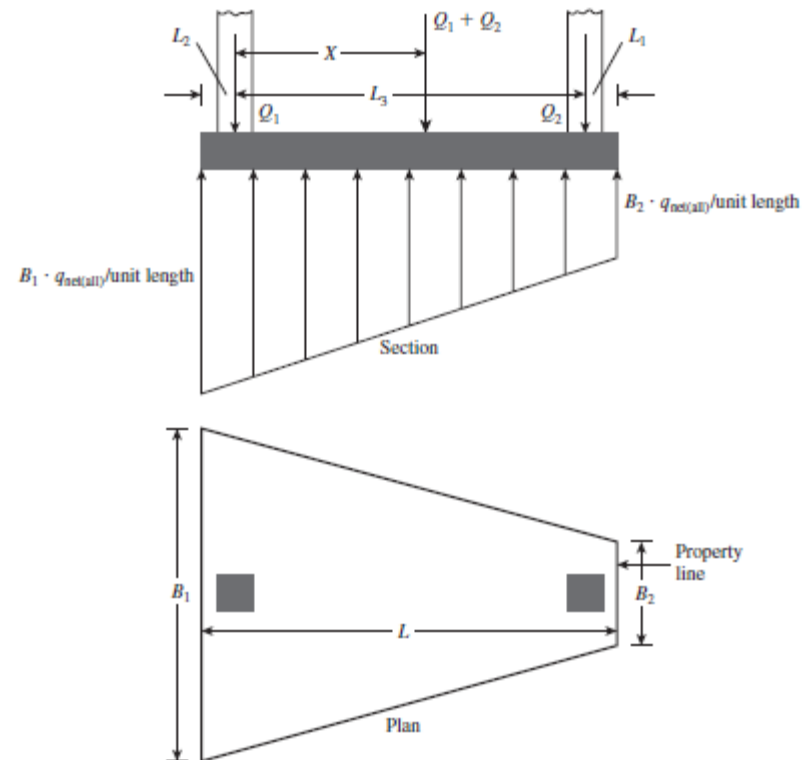
$$0.857 + 1.2 = \left( \frac{B_1 + 2B_2}{B_1 + B_2} \right) \left( \frac{5.2}{3} \right)$$

$$\frac{B_1 + 2B_2}{B_1 + B_2} = 1.187 \quad (b)$$

From Eqs. (a) and (b), we have

$$B_1 = 3.65 \text{ m}$$

$$B_2 = 0.84 \text{ m}$$

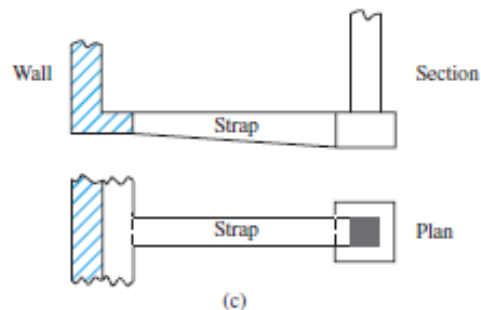
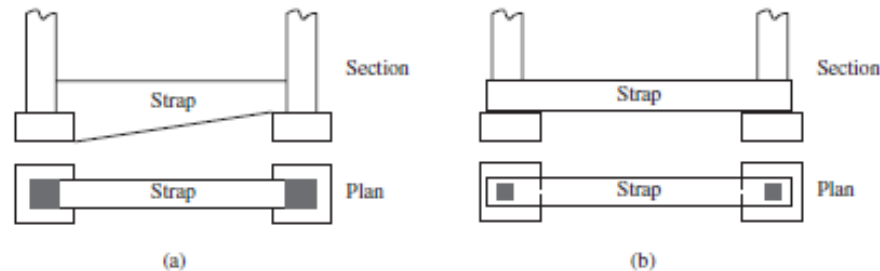


# CANTILEVER FOOTINGS

1. Used when there is a **property line** which prevents the footing to be extended beyond the face of the edge column. In addition to that the edge column is relatively far from the interior column so that the rectangular and trapezoidal combined footings will be **too narrow and long which increases the cost.**
2. May be used to connect two interior foundations, one foundation has a large load require a large area but this area not available, and the other foundation has a small load and there is available area to enlarge this footing, so a strap beam is used to connect these two foundations to transfer the load from the largest to the smallest foundation.
3. There is a “strap beam” which connects two separated footings. The edge footing is usually eccentrically loaded and the interior footing is centrically loaded. **The purpose of the beam is to prevent overturning of the eccentrically loaded footing and to keep uniform pressure under this foundation.**
4. The strap beam doesn't touch the ground (i.e. there is no contact between the strap beam and the soil, so no bearing pressure applied on it).
5. This footing also called **“cantilever footing”** because the overall moment on the strap beam is negative moment.

# CANTILEVER FOOTINGS

- ❑ Cantilever footing construction uses a *strap beam* to connect an eccentrically loaded column foundation to the foundation of an interior column.
- ❑ Cantilever footings may be used in place of trapezoidal or rectangular combined footings when the allowable soil bearing capacity is high and the distances between the columns are large.
- ❑ It consists of two single footings connected with a **beam** or a strap and support two single columns. This type replaces other combined footings and is more economical.



# CANTILEVER FOOTINGS

## Design:

$$R = Q_1 + Q_2 = R_1 + R_2 \quad \text{but, } Q_1 \neq R_1 \text{ and } Q_2 \neq R_2$$

$Q_1$  and  $Q_2$  are knowns but  $R_1$  and  $R_2$  are unknowns

## Finding $X_r$ :

$$\sum M_{Q_1} = 0.0 \text{ (before use of strap beam)} \rightarrow R \times X_r = Q_2 \times d$$

$$a = X_r + \frac{w_1}{2} - \frac{L_1}{2} \quad (L_1 \text{ should be assumed "if not given"})$$

$$b = d - X_r$$

## Finding $R_1$ :

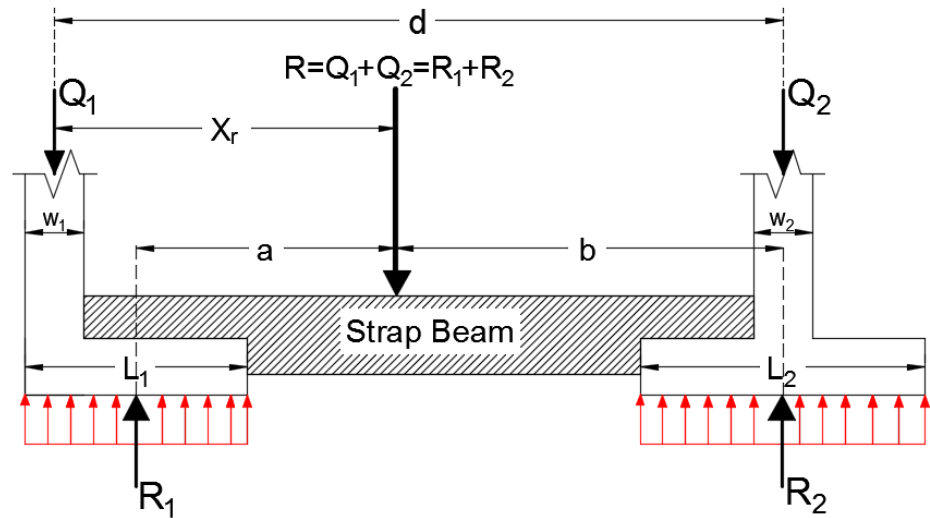
$$\sum M_{R_2} = 0.0 \text{ (after use of strap beam)} \rightarrow R_1 \times (a + b) = R \times b$$

## Finding $R_2$ :

$$R_2 = R - R_1$$

## Design:

$$A_1 = \frac{R_1}{q_{\text{all,net}}}, \quad A_2 = \frac{R_2}{q_{\text{all,net}}}$$



# CANTILEVER FOOTINGS

\* ① If  $\frac{L}{3} < x < \frac{L}{2}$  → Trapezoidal shaped footing.

② If  $x = \frac{L}{2}$  → rectangular combined footing

③ If  $x < \frac{L}{3}$  No solution exists  
 $B_2$  (-ve). B

$$x = \frac{L}{3} \left( \frac{2B_2 + B_1}{B_2 + B_1} \right)$$

If  $x = \frac{L}{3}$  → triangular  $B_2 = 0$

# CANTILEVER FOOTINGS

→ Cases:

① When  $x < \frac{L}{3}$  we need a uniform soil pressure.

or

② When large spacing between two columns which makes it due to the large quantity of concrete in the footing and the high bending moments.

or

③ When the allowable soil bearing capacity is high.

# CANTILEVER FOOTINGS

→ The strap footing design should consider:

a- Strap must be rigid. (to avoid rotation of the exterior footing)

$$\frac{\text{moment of Inertia } I_{\text{strap}}}{I_{\text{footing}}} > 2$$

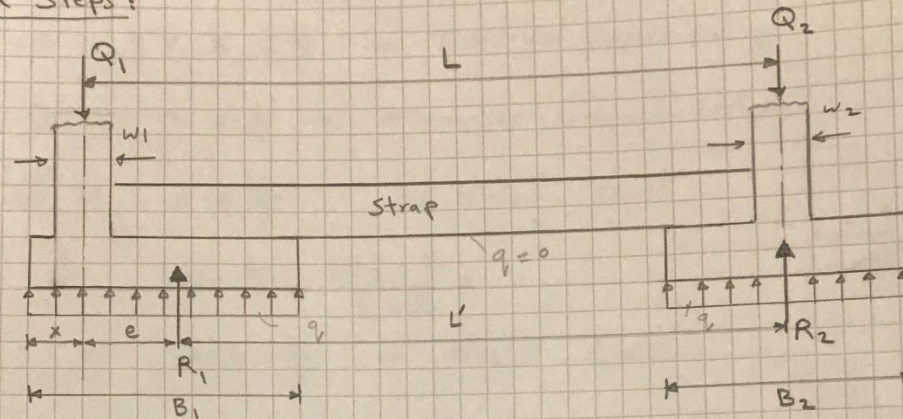
b- Footings should be proportioned for approximately equal soil pressures (Avoid large differences in B to reduce differential settlement).

c- Strap should be out of contact with soil so there are no soil reactions (by loosening the soil beneath the strap, or by leaving open space between strap and soil)

→ strap footing should be the "Last Solution" after doing a careful analysis that spread footings (even if oversized) will not work.

# CANTILEVER FOOTINGS

Design steps:



① Find  $B_1$  by assuming  $e$  or  $B_1$ .

$$B_1 = 2(x+e)$$

② Find the reaction  $R_1$  by taking moments about column 2.  
(you may neglect the weight of strap).

③ Find  $R_2$  from the  $\Sigma F_v = 0$

④ Find  $L_1$  from  $L_1 = \frac{R_1}{B_1 q_{all}}$ .

⑤ Find  $B_2$  from  $B_2 = \sqrt{\frac{R_2}{q_{all}}}$ .

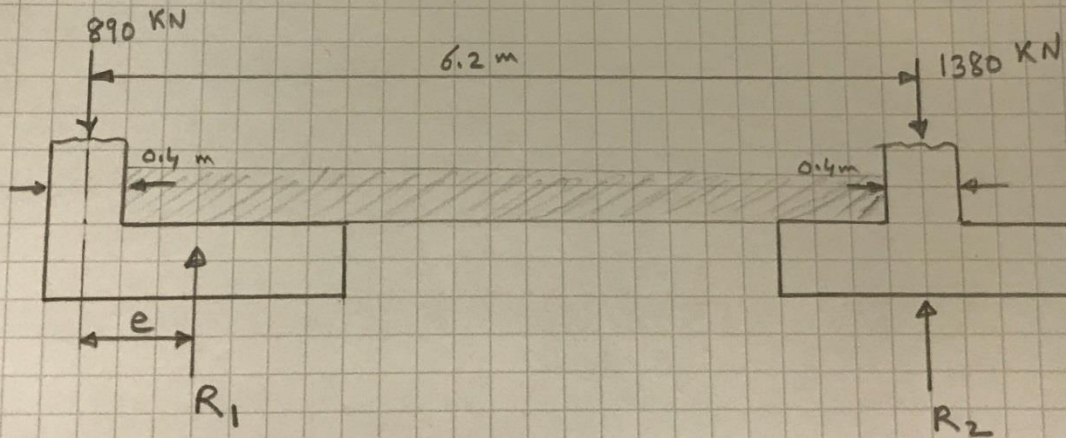
⑥ Find depth,  $A_s$ , strap from shear and moment.



# CANTILEVER FOOTINGS

## Example:

Proportion a strap footing for the conditions show in the Figure below. The allowable soil pressure  $q_a = 120 \text{ KPa}$



# CANTILEVER FOOTINGS

$B_1 \approx \sqrt{\frac{890}{120}} \approx 2.70 \text{ m}$

① Try  $e = 1.2 \text{ m} \rightarrow B_1 = 2(0.2 + 1.2) = 2.8 \text{ m}$

②  $\Sigma M_z = 0$

$$R_1(6.2 - 1.2) - 890(6.20) = 0$$
$$\therefore R_1 = \frac{890(6.20)}{5} = \underline{\underline{1103.6 \text{ KN}}}$$

③  $\Sigma F_y = 0$

$$890 + 1380 - 1103.6 - R_2 = 0$$
$$\therefore R_2 = \underline{\underline{1166.4 \text{ KN}}}$$

④  $L_1 = \frac{R_1}{B_1 q_{\text{all.}}} = \frac{1103.6}{2.8(120)} = 3.28 \text{ m}$

⑤  $B_2 = \sqrt{\frac{R_2}{q_{\text{all.}}}} = \sqrt{\frac{1166.4}{120}} = 3.12 \text{ m}$

<u>Footing 1</u>	<u>Footing 2</u>
$B_1 \times L_1$	$B_2$
$2.80 \times 3.30 \text{ m}$	$3.20 \times 3.20 \text{ m}$

# MAT FOUNDATIONS

Mat foundation is used in the following cases:

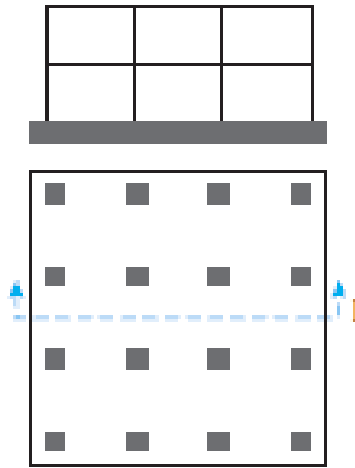
1. If the area of isolated and combined footing  $> 50\%$  of the structure area, because this means the loads are very large and the bearing capacity of the soil is relatively small.
2. If the bearing capacity of the soil is small.
3. If the soil supporting the structure classified as (bad soils) such as:
  - ✓ **Expansive Soil:** Expansive soils are characterized by clayey material that shrinks and swells as it dries or becomes wet respectively. It is recognized from high values of Plasticity Index, Plastic Limit and Shrinkage Limit.
  - ✓ **Compressible soil:** It contains a high content of organic material and not exposed to great pressure during its geological history, so it will be exposed to a significant settlement, so mat foundation is used to avoid differential settlement.
  - ✓ **Collapsible soil:** Collapsible soils are those that appear to be strong and stable in their natural (dry) state, but they rapidly consolidate under wetting, generating large and often unexpected settlements. This can yield disastrous consequences for structures built on such deposits.

# Common Types of Mat Foundations

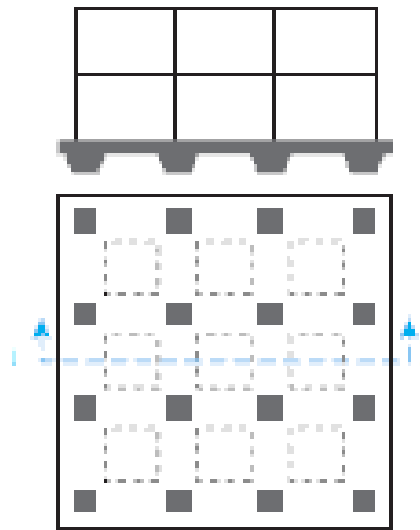
Several types of mat foundations are used currently. Some of the common ones are:

- ❑ **Flat plate. The mat is of uniform thickness.**
- ❑ **Flat plate thickened under columns.**
- ❑ **Beams and slab. The beams run both ways, and the columns are located at the intersection of the beams.**
- ❑ **Flat plates with pedestals.**
- ❑ **Slab with basement walls as a part of the mat. The walls act as stiffeners for the mat.**

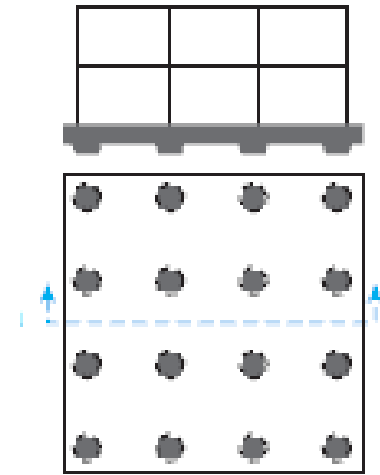
# Common Types of Mat Foundations



Flat plate of uniform thickness

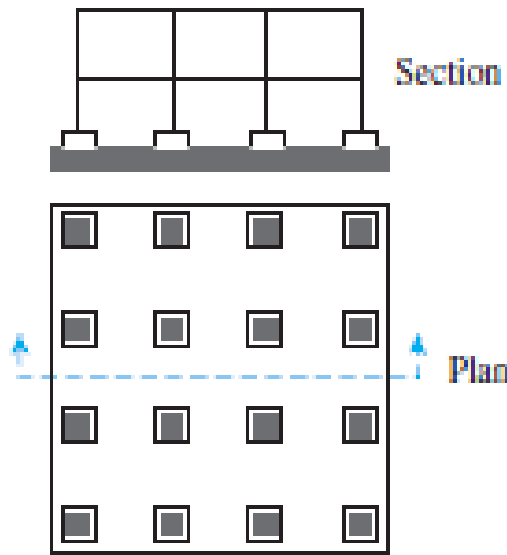


Beams and slab

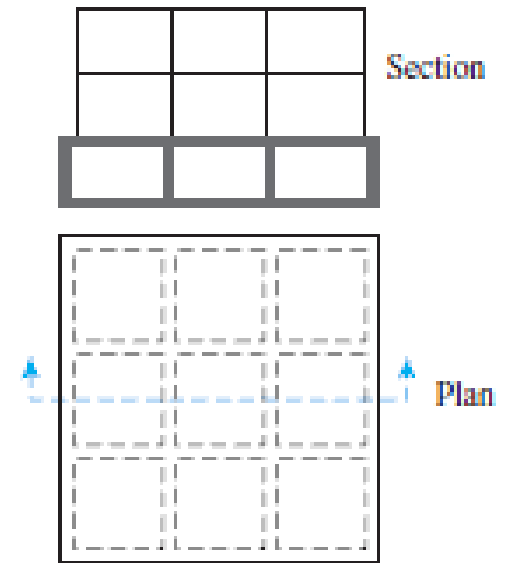


Flat plate thickened under columns

# Common Types of Mat Foundations



**Flat plates with pedestals**

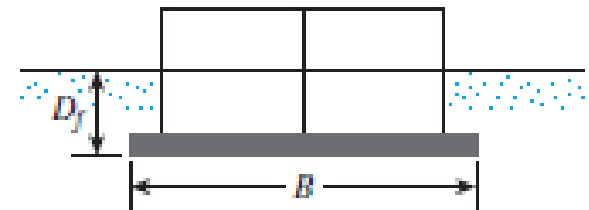
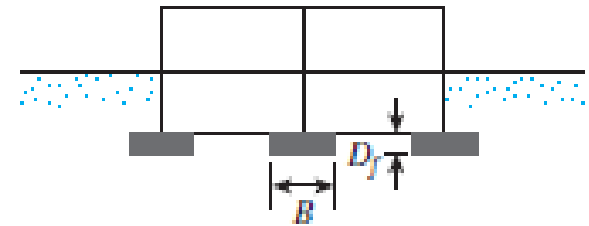


**Slab with basement walls**

# Common Types of Mat Foundations

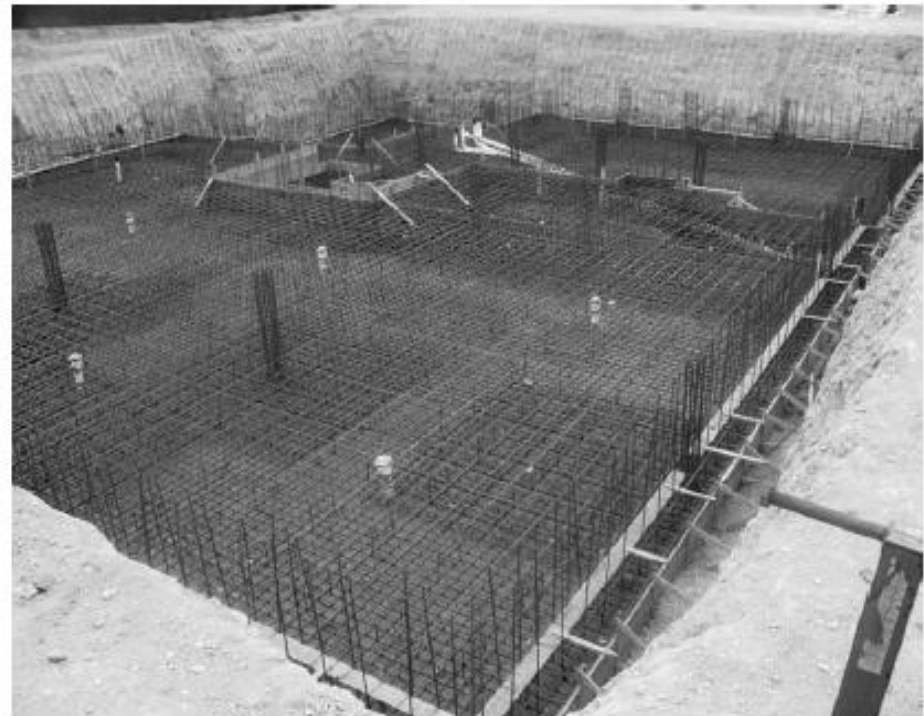
Mats may be supported by piles, which help reduce the settlement of a structure built over highly compressible soil. Where the water table is high, mats are often placed over piles to control buoyancy.

Figure shows the difference between the depth  $D_f$  and the width  $B$  of isolated foundations and mat foundations..



# FLAT-PLATE MAT FOUNDATION

**Flat-plate mat foundation under construction**





# Bearing Capacity of Mat Foundations

The *gross ultimate bearing capacity* of a mat foundation can be determined by

$$q_u = c'N_c F_{cs} F_{cd} F_{c\alpha} + qN_q F_{qs} F_{qd} F_{q\alpha} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma\alpha}$$

The *net ultimate capacity* of a mat foundation

$$q_{net(u)} = q_u - q$$

# Saturated clays with $\phi=0$

$$q_u = c_u N_c F_{cs} F_{cd} + q$$

where  $c_u$  = undrained cohesion. (Note:  $N_c = 5.14$ ,  $N_q = 1$ , and  $N_\gamma = 0$ )  
From Table 4.3, for  $\phi = 0$ ,

$$F_{cs} = 1 + \frac{B}{L} \left( \frac{N_q}{N_c} \right) = 1 + \left( \frac{B}{L} \right) \left( \frac{1}{5.14} \right) = 1 + \frac{0.195B}{L}$$

and

$$F_{cd} = 1 + 0.4 \left( \frac{D_f}{B} \right)$$

Substitution of the preceding shape and depth factors into Eq. (8.10) yields

$$q_u = 5.14c_u \left( 1 + \frac{0.195B}{L} \right) \left( 1 + 0.4 \frac{D_f}{B} \right) + q$$

Hence, the net ultimate bearing capacity is

$$q_{\text{net}(u)} = q_u - q = 5.14c_u \left( 1 + \frac{0.195B}{L} \right) \left( 1 + 0.4 \frac{D_f}{B} \right)$$

For FS = 3, the net allowable soil bearing capacity becomes

$$q_{\text{net(allow)}} = \frac{q_{\text{net}(u)}}{\text{FS}} = 1.713c_u \left( 1 + \frac{0.195B}{L} \right) \left( 1 + 0.4 \frac{D_f}{B} \right)$$

# Granular soils (c=0)

$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08} \left( \frac{B + 0.3}{B} \right)^2 F_d \left( \frac{S_e}{25} \right)$$

where

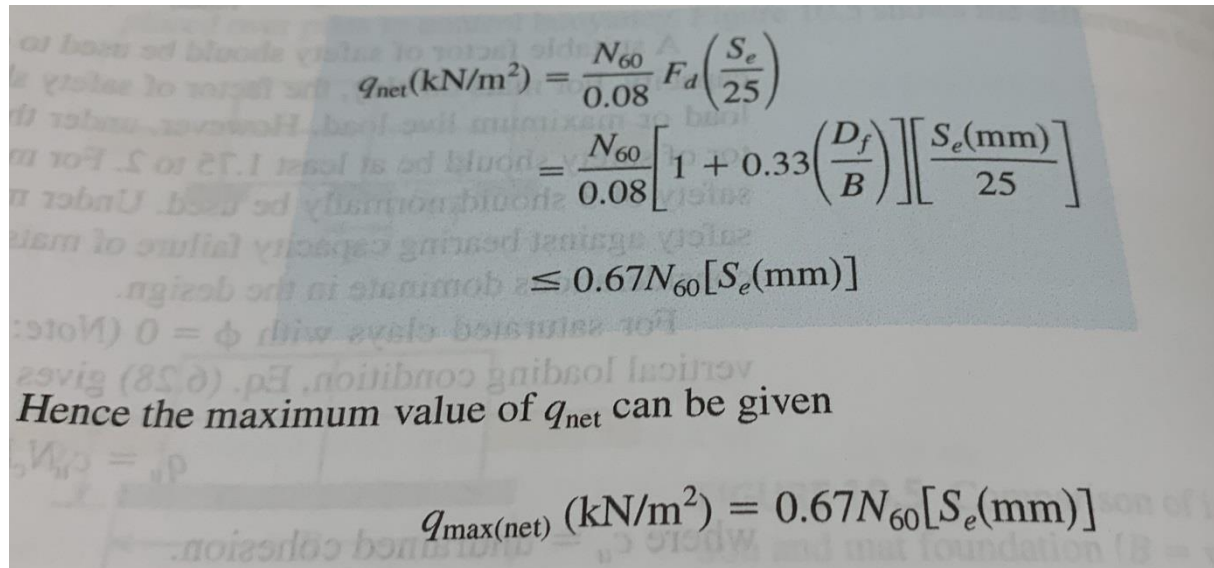
$N_{60}$  = standard penetration resistance

$B$  = width (m)

$F_d = 1 + 0.33(D_f/B) \leq 1.33$

$S_e$  = settlement, (mm)

When the width  $B$  is large, the preceding equation can be approximated as


$$q_{\text{net}}(\text{kN/m}^2) = \frac{N_{60}}{0.08} F_d \left( \frac{S_e}{25} \right)$$
$$= \frac{N_{60}}{0.08} \left[ 1 + 0.33 \left( \frac{D_f}{B} \right) \right] \left[ \frac{S_e(\text{mm})}{25} \right]$$
$$\leq 0.67 N_{60} [S_e(\text{mm})]$$

Hence the maximum value of  $q_{\text{net}}$  can be given

$$q_{\text{max(net)}} (\text{kN/m}^2) = 0.67 N_{60} [S_e(\text{mm})]$$

# Net pressure on soil caused by a mat foundation

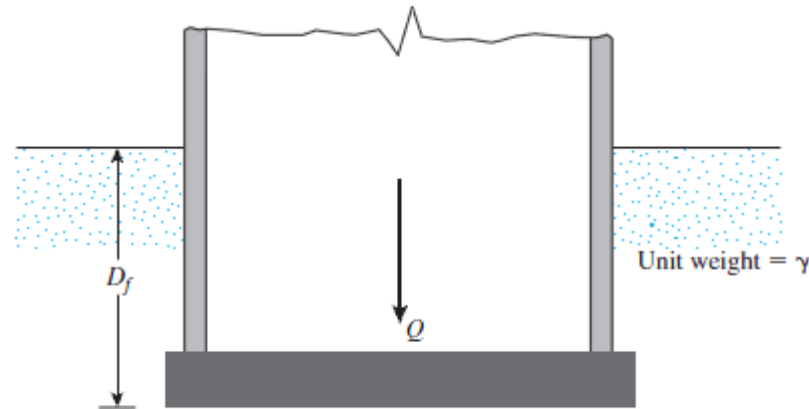
$$q = \frac{Q}{A} - \gamma D_f$$

where

$Q$  = dead weight of the structure and the live load

$A$  = area of the raft

In all cases,  $q$  should be less than or equal to allowable  $q_{\text{net(all)}}$ .



**Figure 8.7** Definition of net pressure on soil caused by a mat foundation

## EXAMPLE 10.3

Determine the net ultimate bearing capacity of a mat foundation measuring  $20 \text{ m} \times 8 \text{ m}$  on a saturated clay with  $c_u = 85 \text{ kN/m}^2$ ,  $\phi = 0$ , and  $D_f = 1.5 \text{ m}$ .

### Solution

From Eq. (8.12),

$$\begin{aligned} q_{\text{net}(u)} &= 5.14c_u \left[ 1 + \left( \frac{0.195B}{L} \right) \right] \left[ 1 + 0.4 \frac{D_f}{B} \right] \\ &= (5.14)(85) \left[ 1 + \left( \frac{0.195 \times 8}{20} \right) \right] \left[ 1 + \left( \frac{0.4 \times 1.5}{8} \right) \right] \\ &= 506.3 \text{ kN/m}^2 \end{aligned}$$



# EXAMPLE 10.4

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**Read Example 10.4**

# Compensated Foundation

Net average applied pressure on soil is

$$q = \frac{Q}{A} - \gamma D_f$$

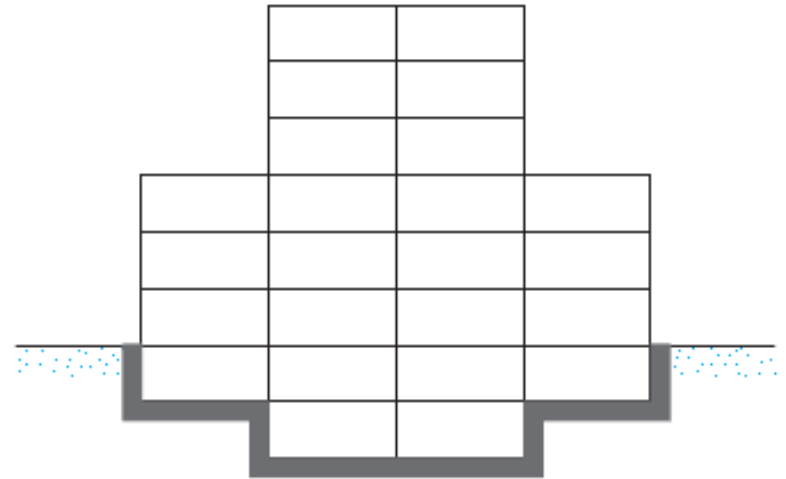
For no increase in the net pressure on soil below a mat foundation,  $q$  should be zero. Thus,

$$D_f = \frac{Q}{A\gamma}$$

$D_f$  = the depth of a fully compensated foundation

$$FS = \frac{q_{net(u)}}{q} = \frac{q_{net(u)}}{\frac{Q}{A} - \gamma D_f}$$

where  $q_{net(u)}$  = net ultimate bearing capacity



For saturated clays

$$FS = \frac{5.14c_u \left(1 + \frac{0.195B}{L}\right) \left(1 + 0.4 \frac{D_f}{B}\right)}{\frac{Q}{A} - \gamma D_f}$$

# EXAMPLE 10.5

The mat shown in Figure 8.7 has dimensions of 20 m  $\times$  30 m. The total dead and live load on the mat is 110 MN. The mat is placed over a saturated clay having a unit weight of 18 kN/m<sup>3</sup> and  $c_u = 140$  kN/m<sup>2</sup>. Given that  $D_f = 1.5$  m, determine the factor of safety against bearing capacity failure.

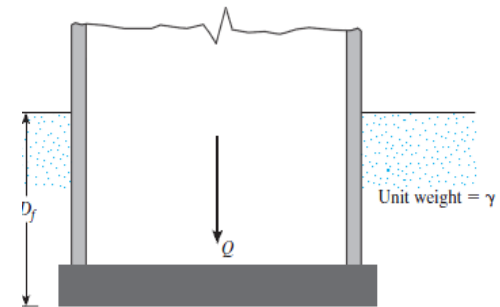
## Solution

From Eq. (8.23), the factor of safety

$$FS = \frac{5.14c_u \left(1 + \frac{0.195B}{L}\right) \left(1 + 0.4 \frac{D_f}{B}\right)}{\frac{Q}{A} - \gamma D_f}$$

We are given that  $c_u = 140$  kN/m<sup>2</sup>,  $D_f = 1.5$  m,  $B = 20$  m,  $L = 30$  m, and  $\gamma = 18$  kN/m<sup>3</sup>. Hence,

$$FS = \frac{(5.14)(140) \left[1 + \frac{(0.195)(20)}{30}\right] \left[1 + 0.4 \left(\frac{1.5}{20}\right)\right]}{\left(\frac{110,000 \text{ kN}}{20 \times 30}\right) - (18)(1.5)} = 5.36$$

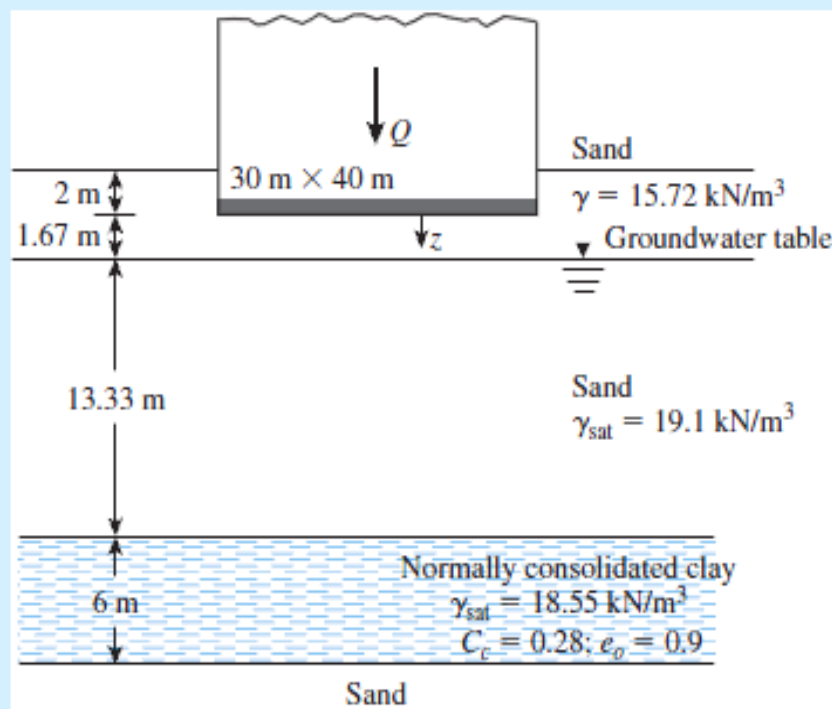


3.7 Definition of net pressure on soil caused by a mat foundation



# EXAMPLE 10.6

Consider a mat foundation  $30\text{ m} \times 40\text{ m}$  in plan, as shown in Figure 8.9. The total dead load and live load on the raft is  $200 \times 10^3\text{ kN}$ . Estimate the consolidation settlement at the center of the foundation.



**Figure 8.9** Consolidation settlement under a mat foundation

# EXAMPLE 10.6

## Solution

From Eq. (2.65)

$$S_{c(\varphi)} = \frac{C_c H_c}{1 + e_o} \log \left( \frac{\sigma'_o + \Delta\sigma'_{av}}{\sigma'_o} \right)$$

$$\sigma'_o = (3.67)(15.72) + (13.33)(19.1 - 9.81) + \frac{6}{2}(18.55 - 9.81) \approx 208 \text{ kN/m}^2$$

$$H_c = 6 \text{ m}$$

$$C_c = 0.28$$

$$e_o = 0.9$$

For  $Q = 200 \times 10^3$  kN, the net load per unit area is

$$q = \frac{Q}{A} - \gamma D_f = \frac{200 \times 10^3}{30 \times 40} - (15.72)(2) \approx 135.2 \text{ kN/m}^2$$

In order to calculate  $\Delta\sigma'_{av}$  we refer to Section 6.8. The loaded area can be divided into four areas, each measuring 15 m  $\times$  20 m. Now using Eq. (6.23), we can calculate the average stress increase in the clay layer below the corner of each rectangular area, or

$$\Delta\sigma'_{av(H_2/H_1)} = q_o \left[ \frac{H_2 I_{a(H_2)} - H_1 I_{a(H_1)}}{H_2 - H_1} \right]$$

## EXAMPLE 10.6

For  $I_{a(H_2)}$ ,

$$m_2 = \frac{B}{H_2} = \frac{15}{1.67 + 13.33 + 6} = 0.71$$
$$n_2 = \frac{L}{H_2} = \frac{20}{21} = 0.95$$

From Fig. 6.11, for  $m_2 = 0.71$  and  $n_2 = 0.95$ , the value of  $I_{a(H_2)}$  is 0.21. Again, for  $I_{a(H_1)}$ ,

$$m_2 = \frac{B}{H_1} = \frac{15}{15} = 1$$
$$n_2 = \frac{L}{H_1} = \frac{20}{15} = 1.33$$

From Figure 6.11,  $I_{a(H_1)} = 0.225$ , so

$$\Delta\sigma'_{av(H_2/H_1)} = 135.2 \left[ \frac{(21)(0.21) - (15)(0.225)}{6} \right] = 23.32 \text{ kN/m}^2$$

So, the stress increase below the center of the  $30 \text{ m} \times 40 \text{ m}$  area is  $(4)(23.32) = 93.28 \text{ kN/m}^2$ . Thus

$$S_{c(p)} = \frac{(0.28)(6)}{1 + 0.9} \log \left( \frac{208 + 93.28}{208} \right) = 0.142 \text{ m}$$
$$= 142 \text{ mm}$$

# Structural Design of Mat Foundations

The structural design of mat foundations can be carried out by the following methods:

- The conventional rigid method
- The approximate flexible method.
- Finite-difference and finite-element methods

# The Conventional Rigid Method

Step 1. Figure 8.10a shows mat dimensions of  $L \times B$  and column loads of  $Q_1, Q_2, Q_3, \dots$ . Calculate the total column load as

$$Q = Q_1 + Q_2 + Q_3 + \dots \quad (8.24)$$

Step 2. Determine the pressure on the soil,  $q$ , below the mat at points  $A, B, C, D, \dots$ , by using the equation

$$q = \frac{Q}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x} \quad (8.25)$$

where

$$A = BL$$

$$I_x = (1/12)BL^3 = \text{moment of inertia about the } x\text{-axis}$$

$$I_y = (1/12)LB^3 = \text{moment of inertia about the } y\text{-axis}$$

$$M_x = \text{moment of the column loads about the } x\text{-axis} = Qe_y$$

$$M_y = \text{moment of the column loads about the } y\text{-axis} = Qe_x$$

The load eccentricities,  $e_x$  and  $e_y$ , in the  $x$  and  $y$  directions can be determined by using  $(x', y')$  coordinates:

$$x' = \frac{Q_1 x'_1 + Q_2 x'_2 + Q_3 x'_3 + \dots}{Q} \quad (8.26)$$

and

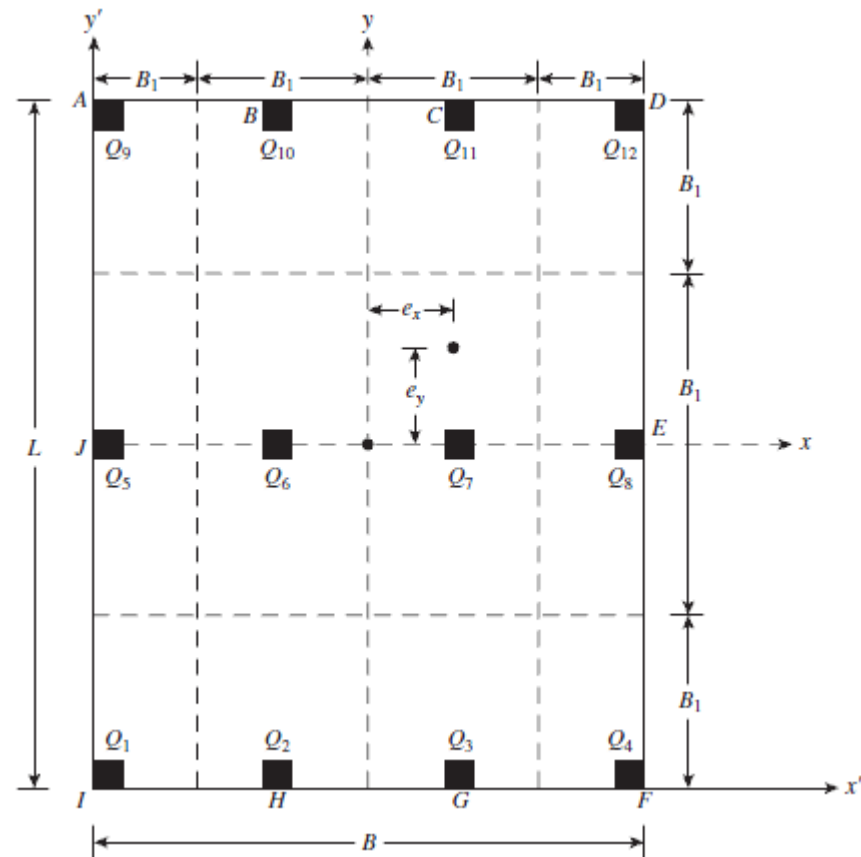
$$e_x = x' - \frac{B}{2} \quad (8.27)$$

Similarly,

$$y' = \frac{Q_1 y'_1 + Q_2 y'_2 + Q_3 y'_3 + \dots}{Q} \quad (8.28)$$

and

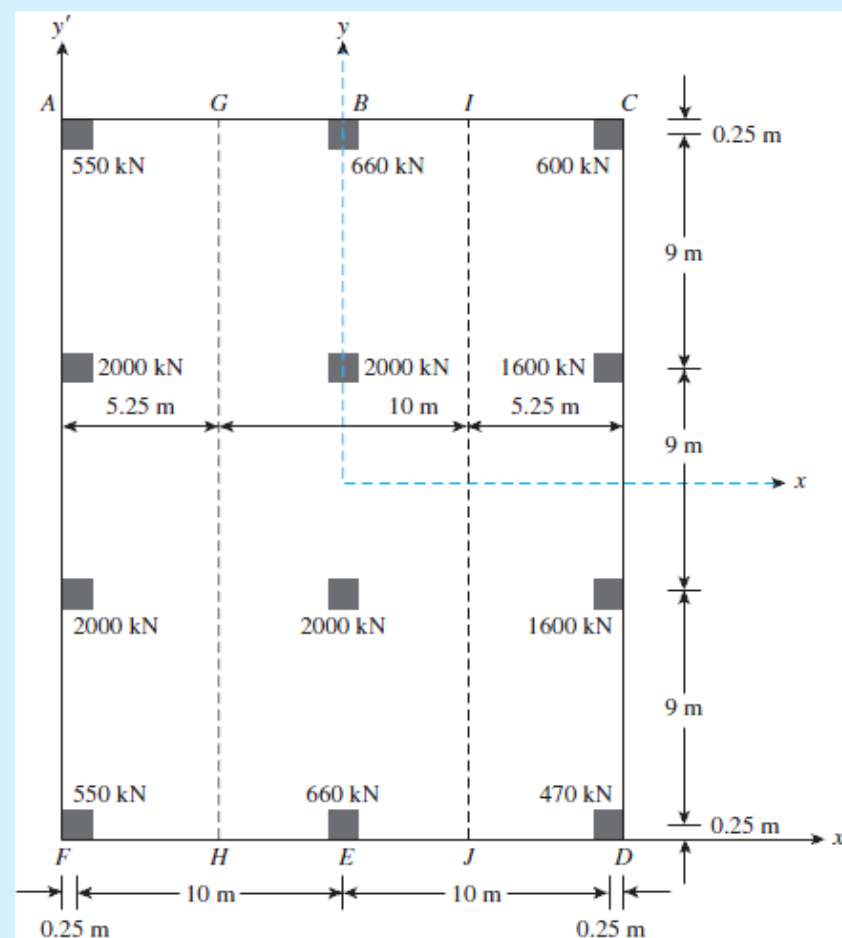
$$e_y = y' - \frac{L}{2} \quad (8.29)$$



Step 3. Compare the values of the soil pressures determined in Step 2 with the net allowable soil pressure to determine whether  $q \leq q_{all(net)}$ .

# EXAMPLE 10.7

The plan of a mat foundation is shown in Figure 8.14. Calculate the soil pressure at points *A*, *B*, *C*, *D*, *E*, and *F*. (Note: All column sections are planned to be 0.5 m × 0.5 m.) All loads shown are factored loads according to ACI 381-11 (2011).



# EXAMPLE 10.7

## Solution

$$q = \frac{Q}{A} \pm \frac{M_y x}{I_y} \pm \frac{M_x y}{I_x}$$

$$A = (20.5)(27.5) = 563.75 \text{ m}^2$$

$$I_x = \frac{1}{12}BL^3 = \frac{1}{12}(20.5)(27.5)^3 = 35,528 \text{ m}^4$$

$$I_y = \frac{1}{12}LB^3 = \frac{1}{12}(27.5)(20.5)^3 = 19,743 \text{ m}^4$$

$$Q = 470 + (2)(550) + 600 + (2)(660) + (2)(1600) + (4)(2000) = 14,690 \text{ kN}$$

$$M_y = Qe_x; \quad e_x = x' - \frac{B}{2}$$

$$x' = \frac{Q_1 x'_1 + Q_2 x'_2 + Q_3 x'_3 + \dots}{Q}$$

$$= \frac{1}{14,690} \left[ \begin{array}{l} (10.25)(660 + 2000 + 2000 + 660) \\ + (20.25)(470 + 1600 + 1600 + 600) \\ + (0.25)(550 + 2000 + 2000 + 550) \end{array} \right] = 9.686 \text{ m}$$

$$e_x = x' - \frac{B}{2} = 9.686 - 10.25 = -0.565 \text{ m} \approx -0.57 \text{ m}$$

# EXAMPLE 10.7

Hence, the resultant line of action is located to the left of the center of the mat. So  $M_y = (14,690)(0.57) = 8373$  kN-m. Similarly

$$M_x = Qe_y; \quad e_y = y' - \frac{L}{2}$$

$$y' = \frac{Q_1y'_1 + Q_2y'_2 + Q_3y'_3 + \dots}{Q}$$

$$= \frac{1}{14,690} \left[ (0.25)(550 + 660 + 470) + (9.25)(2000 + 2000 + 1600) \right. \\ \left. + (18.25)(2000 + 2000 + 1600) + (27.25)(550 + 660 + 600) \right]$$

$$= 13.86 \text{ m}$$

$$e_y = y' - \frac{L}{2} = 13.86 - 13.75 = 0.11 \text{ m}$$

The location of the line of action of the resultant column loads is shown in Figure

$$M_x = (14,690)(0.11) = 1616 \text{ kN-m. So}$$

$$q = \frac{14,690}{563.75} \pm \frac{8373x}{19743} \pm \frac{1616y}{35,528} = 26.0 \pm 0.42x \pm 0.05y \text{ (kN/m}^2\text{)}$$

Therefore,

$$\text{At A: } q = 26 + (0.42)(10.25) + (0.05)(13.75) = 31.0 \text{ kN/m}^2$$

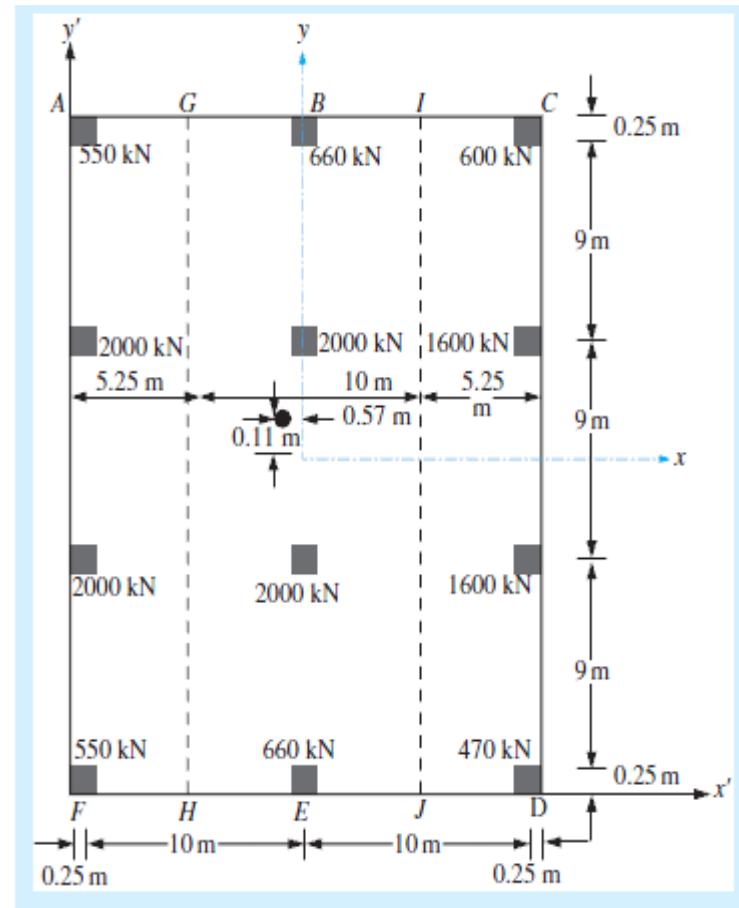
$$\text{At B: } q = 26 + (0.42)(0) + (0.05)(13.75) = 26.68 \text{ kN/m}^2$$

$$\text{At C: } q = 26 - (0.42)(10.25) + (0.05)(13.75) = 22.38 \text{ kN/m}^2$$

$$\text{At D: } q = 26 - (0.42)(10.25) - (0.05)(13.75) = 21.0 \text{ kN/m}^2$$

$$\text{At E: } q = 26 + (0.42)(0) - (0.05)(13.75) = 25.31 \text{ kN/m}^2$$

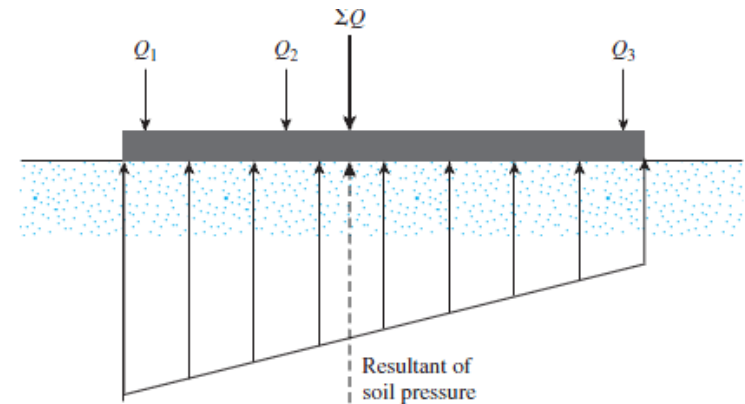
$$\text{At F: } q = 26 + (0.42)(10.25) - (0.05)(13.75) = 29.61 \text{ kN/m}^2$$



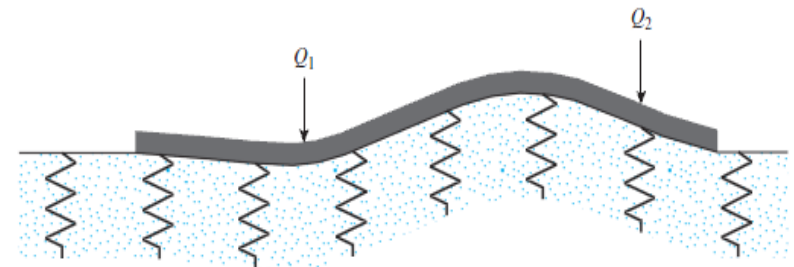


# Design of Mat Foundations

In the **conventional rigid method** of design, the mat is assumed to be infinitely rigid. Also, the soil pressure is distributed in a straight line, and the centroid of the soil pressure is coincident with the line of action of the resultant column loads.



In the **approximate flexible method** of design, the soil is assumed to be equivalent to an infinite number of elastic springs, as shown in the figure. This assumption is sometimes referred to as the **Winkler foundation**. The elastic constant of these assumed springs is referred to as the **coefficient of subgrade reaction,  $k$** .



# Design of Mat Foundations

$$\beta = \sqrt[4]{\frac{B_1 k}{4E_F I_F}}$$

This parameter is very important in determining whether a mat foundation should be designed by the conventional rigid method or the approximate flexible method.

According to the American Concrete Institute Committee 336 (1988), mats should be designed by the conventional rigid method if the spacing of columns in a strip is less than  $1.75/\beta$ .

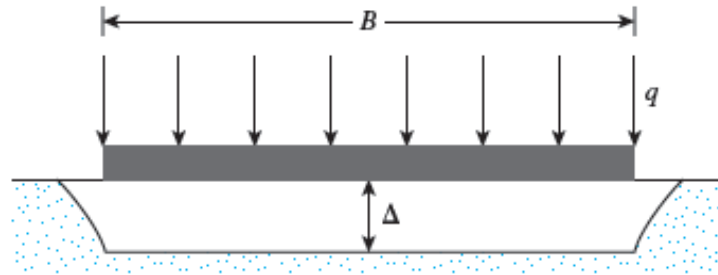
If the spacing of columns is larger than  $1.75/\beta$ , the approximate flexible method may be used.

# The coefficient of subgrade reaction

If a foundation of width  $B$  is subjected to a load per unit area of  $q$ , it will undergo a settlement  $D$ . The coefficient of subgrade reaction can be defined as:

$$k = \frac{q}{\Delta}$$

The unit of  $k$  is  $\text{kN/m}^3$



The value of the coefficient of subgrade reaction is not a constant for a given soil, but rather depends on several factors, such as the length  $L$  and width  $B$  of the foundation and also the depth of embedment of the foundation.

A comprehensive study by Terzaghi (1955) of the parameters affecting the coefficient of subgrade reaction indicated that the value of the coefficient decreases with the width of the foundation.

In the field, load tests can be carried out by means of square plates measuring  $0.3 \text{ m} \times 0.3 \text{ m}$ , and values of  $k$  can be calculated. The value of  $k$  can be related to large foundations measuring  $B \times B$ .

# The coefficient of subgrade reaction

## Foundations on Sandy Soils

$$k = k_{0.3} \left( \frac{B + 0.3}{2B} \right)^2$$

## Foundations on Clays

$$k(\text{kN/m}^3) = k_{0.3}(\text{kN/m}^3) \left[ \frac{0.3 \text{ (m)}}{B \text{ (m)}} \right]$$

where  $k_{0.3}$  and  $k$  = coefficients of subgrade reaction of foundations measuring  $0.3 \text{ m} \times 0.3 \text{ m}$  and  $B \text{ (m)} \times B \text{ (m)}$ , respectively (unit is  $\text{kN/m}^3$ ).

# The coefficient of subgrade reaction

For rectangular foundations having dimensions of  $B \times L$  :

$$k = \frac{k_{(B \times B)} \left( 1 + 0.5 \frac{B}{L} \right)}{1.5}$$

$k$  = coefficient of subgrade reaction of the rectangular foundation ( $L \times B$ )

$k_{(B \times B)}$  = coefficient of subgrade reaction of a square foundation having dimension of  $B \times B$

The value of  $k$  for a very long foundation with a width  $B$  is approximately  $0.67k_{(B \times B)}$

$$k = \frac{E_s}{B(1 - \mu_s^2)}$$

$E_s$  = modulus of elasticity of soil

$B$  = foundation width

$\mu_s$  = Poisson's ratio of soil

# The coefficient of subgrade reaction

**TABLE 10.2** Typical Subgrade Reaction Values,  $k_{0.3}$

Soil type	$k_{0.3}$ (MN/m <sup>3</sup> )
Dry or moist sand:	
Loose	8–25
Medium	25–125
Dense	125–375
Saturated sand:	
Loose	10–15
Medium	35–40
Dense	130–150
Clay:	
Stiff	10–25
Very stiff	25–50
Hard	>50

*The end*